MALL Proof Nets: weights vs abstraction and efficiency

(a summary of the state of the art)

Roberto Maieli

Università degli Studi "Roma Tre"
maieli@uniroma3.it

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The MALL proof nets renaissance

- Since its inception (1987) the problem of finding a "good notion" of MALL proof nets has remained open.
- Last few years have seen a renaissance of this theme:
  - (1996, Girard, Monomial Nets)
  - 2003, Hughes-van Glabbeek, Linkings Nets
  - 2004, Hamano, ext. of Mon. MALL PN (mix, softness analys.)
  - 2004, Laurent-Tortora, (slice) normaliz. for pol. MALL PN
  - 2005, Curien-Faggian, L-nets
  - 2007, Maieli, Contractible MALL PN
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  - 2008, Mogbil-de Naurois. Correctness of MALL is PS NL-Complete
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- here a comparing of the current technologies for MALL PN, based on "weights" (*dependencies*) w.r.t.

  abstraction and **efficiency** of the representation.

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Standard key ingredients of a Proof Nets (PN) syntax

naively

- PN are parallel presentations of sequential proofs (SP) of LL
- PN quotient classes of equivalent proofs, modulo irrelevant permutations of derivation rules.

key ingredients:

- a graph syntax (proof structures, PSs)
- a correctness criterion (defining *PNs* among PSs)
- an interpretation of the sequent calculus proofs (SPs)
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Main properties for a "good notion" of PN (1/3)

The **interpretation** (translation) of SP into PS must be:

Sound: the PS associated to a SP, must be *correct* (a PN);

Function: $SP \mapsto PN$;

Canonical surjection: SP equal up to (reasonable) commutations of rules must be identified upon translation to a PN;

Efficient: P-time in the size (of the proofs).

(naively) we should preserve the computational complexity of the interpreted proofs;

(seriously) we should respect the notion that a *semantics* (PN) is a *structure-preserving map* or some kind of *homomorphism* from proofs.

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The **cut elimination** procedure must be

Defined directly on PS;

Complete: any cut node of a PS reduces in one step;

Local: a cut elimination step only affects the nodes (immediately) connected to the reducing cut node;

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Main properties for a “good notion” of PN (3/3)

Finally, the **correctness criterion** must be:

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MALL Proof Structures: *weights*

- the **problem** is to cope with the &-rule

$$\frac{\Gamma, A \qquad \Gamma, B}{\Gamma, A \& B}\,\&$$

for which a **superimposition** of two proof nets must be made.

MALL Proof Structures: *weights*

- A **solution**: a **boolean variable** (*eigen-wight*) for each &-link:

$$\frac{\Gamma, A \qquad \Gamma, B}{\Gamma, A \& B} \&_{p}$$

MALL Proof Structures: *weights*

- which separates the two **slices** of the superimposition:

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MALL Proof Structures: *weights*

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$$\bar{p} \; \textit{slice}$$

$$\frac{\Gamma, B}{\Gamma, A \& B} \, \&_p$$

MALL Proof Structures: *weights*

- then, we can get different notions of PN in which links are *weighted* by non-zero:
  - **monomials** (Girard, 1998)

    dependence condition:
    if L depends on p then $w(L) \leq w(\&_p)$
  - or (general) **polynomials** ($\sim$ Hughes-Van Glabbeek, 2003).

    no dependence at all!

  of the *Bool*-algebra generated by the eigen weights.

Interpretation: MALL SP $\mapsto$ Monomial PN (1/7)

- There is **no canonical surjection** from SP to Monomial PN.
- There is only a non-surjective mapping allowing a *minimal* (only on conclusion links) superimposition of slices

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Example

$$\dfrac{\dfrac{\dfrac{}{B^\perp, B}\,\text{ax} \qquad \dfrac{\dfrac{}{A^\perp, A}\,\text{ax} \qquad \dfrac{}{A^\perp, A}\,\text{ax}}{A^\perp, A \& A}\,\&}{B^\perp, B \otimes A^\perp, A \& A}\,\otimes \qquad \dfrac{\dfrac{\dfrac{}{B^\perp, B}\,\text{ax} \qquad \dfrac{}{A^\perp, A}\,\text{ax}}{B^\perp, B \otimes A^\perp, A}\,\otimes \qquad \dfrac{\dfrac{}{B^\perp, B}\,\text{ax} \qquad \dfrac{}{A^\perp, A}\,\text{ax}}{B^\perp, B \otimes A^\perp, A}\,\otimes}{B^\perp, B \otimes A^\perp, A \& A}\,\&}{B^\perp \& B^\perp, B \otimes A^\perp, A \& A}\,\&$$

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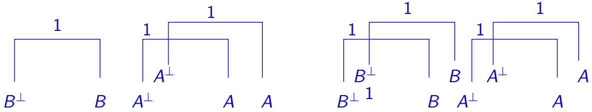
$$
\dfrac{
  \dfrac{\dfrac{}{B^\perp, B}\,\mathsf{ax} \qquad \dfrac{\dfrac{}{A^\perp, A}\,\mathsf{ax} \quad \dfrac{}{A^\perp, A}\,\mathsf{ax}}{A^\perp, A \& A}}{B^\perp, B \otimes A^\perp, A \& A}
  \qquad
  \dfrac{\dfrac{\dfrac{}{B^\perp, B}\,\mathsf{ax} \quad \dfrac{}{A^\perp, A}\,\mathsf{ax}}{B^\perp, B \otimes A^\perp, A} \quad \dfrac{\dfrac{}{B^\perp, B}\,\mathsf{ax} \quad \dfrac{}{A^\perp, A}\,\mathsf{ax}}{B^\perp, B \otimes A^\perp, A}}{B^\perp, B \otimes A^\perp, A \& A}
}{B^\perp \& B^\perp, B \otimes A^\perp, A \& A}
$$

axioms map to



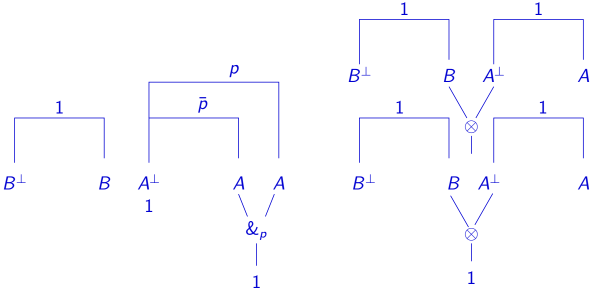
Interpretation: MALL SP $\mapsto$ Monomial PN (3/7)

$$\dfrac{\dfrac{}{B^\perp, B} \qquad \dfrac{\dfrac{}{A^\perp, A} \quad \dfrac{}{A^\perp, A}}{A^\perp, A \& A}\,\&}{B^\perp, B \otimes A^\perp, A \& A} \qquad \dfrac{\dfrac{\dfrac{}{B^\perp, B} \quad \dfrac{}{A^\perp, A}}{B^\perp, B \otimes A^\perp, A}\,\otimes \qquad \dfrac{\dfrac{}{B^\perp, B} \quad \dfrac{}{A^\perp, A}}{B^\perp, B \otimes A^\perp, A}\,\otimes}{B^\perp, B \otimes A^\perp, A \& A}$$

$$\overline{\hspace{20em}}$$

$$B^\perp \& B^\perp, B \otimes A^\perp, A \& A$$

the topmost $\&$ (with eigen-weight p) and $\otimes$-rules map to



Interpretation: MALL SP $\mapsto$ Monomial PN (4/7)

$$
\dfrac{
  \dfrac{
    \dfrac{}{B^\perp, B}
    \quad
    \dfrac{\dfrac{}{A^\perp, A} \quad \dfrac{}{A^\perp, A}}{A^\perp, A\&A}
  }{B^\perp, B \otimes A^\perp, A\&A}\otimes
  \qquad
  \dfrac{
    \dfrac{\dfrac{}{B^\perp, B} \quad \dfrac{}{A^\perp, A}}{B^\perp, B \otimes A^\perp, A}
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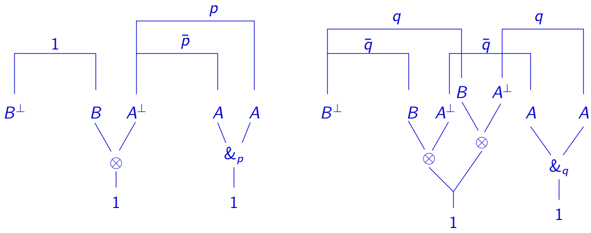
$$

the middle $\otimes$ and &-rules (with eigen-weight q) map to:



Interpretation: MALL SP $\mapsto$ Monomial PN (5/7)

$$
\dfrac{
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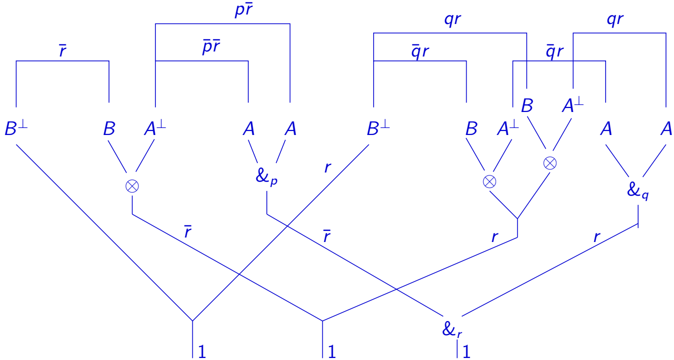
$$

finally, the lowest &-rule (with eigen-weight r) maps to:



Interpretation: MALL SP $\mapsto$ Monomial PN (6/7)

It is **not invariant** under the raising of the $⅋, \otimes, \oplus, \&$ over the $\&$-rule

Example: if we permute the $\otimes$ over the $\&_p$-rule

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Interpretation: MALL SP $\mapsto$ Monomial PN (7/8)

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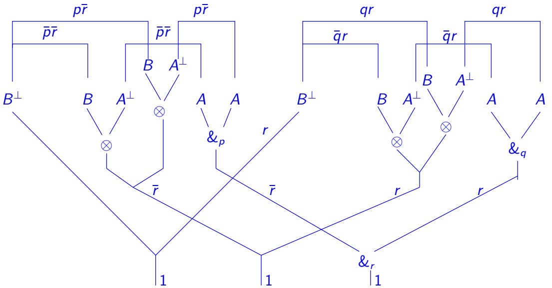
then, this SP maps into a different (w.r.t. the previous one) PN:



Interpretation: MALL SP $\mapsto$ Monomial PN (8/8)

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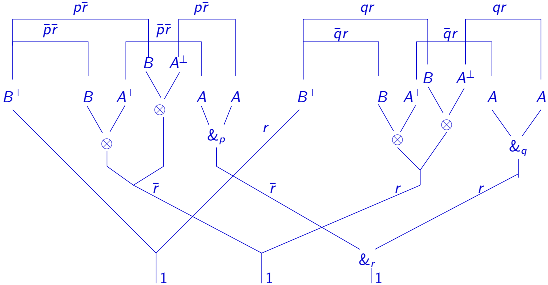

when we add a &-link, we don't know if a link L_1 of π_1 is the same as another link L' of π_2;

in general, $p.w_1(L) + \bar{p}.w_2(L_2)$ is not a monomial, except when L_1, L_2 are conclusions

Interpretation: MALL SP $\mapsto$ Polynomial PN (1/8)

There is **a canonical surjection** from MALL SP to Polynomial PN

Interpretation: MALL SP $\mapsto$ Polynomial PN (2/8)

There is **a canonical surjection** from MALL SP to Polynomial PN

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assign an eigen weight to each & in the sequent conclusion and (upwards) propagate them

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\dfrac{\dfrac{\dfrac{\bar{q}}{B^\perp, B} \quad \dfrac{\bar{q}}{A^\perp, A}}{B^\perp, B \otimes A^\perp, A} \quad \dfrac{\dfrac{q}{B^\perp, B} \quad \dfrac{q}{A^\perp, A}}{B^\perp, B \otimes A^\perp, A}}{B^\perp, B \otimes A^\perp, A \&_{\color{blue}q} A}\,{\color{blue}\&_q}
}{B^\perp \&_{\color{red}p} B^\perp, B \otimes A^\perp, A \&_{\color{blue}q} A}
$$

inductively (top-down) separate each slice with monomials

Interpretation: MALL SP $\mapsto$ Polynomial PN (5/8)

There is **a canonical surjection** from MALL SP to Polynomial PN

Example:

$$
\dfrac{
\dfrac{\dfrac{\bar{p}}{B^\perp, B} \qquad \dfrac{\dfrac{\bar{p}\bar{q}}{A^\perp, A} \quad \dfrac{\bar{p}q}{A^\perp, A}}{A^\perp, A \&_q A}\&}{B^\perp, B \otimes A^\perp, A \&_q A}
\qquad
\dfrac{\dfrac{\dfrac{p\bar{q}}{B^\perp, B} \quad \dfrac{p\bar{q}}{A^\perp, A}}{B^\perp, B \otimes A^\perp, A}\otimes \quad \dfrac{\dfrac{pq}{B^\perp, B} \quad \dfrac{pq}{A^\perp, A}}{B^\perp, B \otimes A^\perp, A}\otimes}{B^\perp, B \otimes A^\perp, A \&_q A}\&
}{B^\perp \&_p B^\perp, B \otimes A^\perp, A \&_q A}\&_p
$$

Interpretation: MALL SP $\mapsto$ Polynomial PN (6/8)

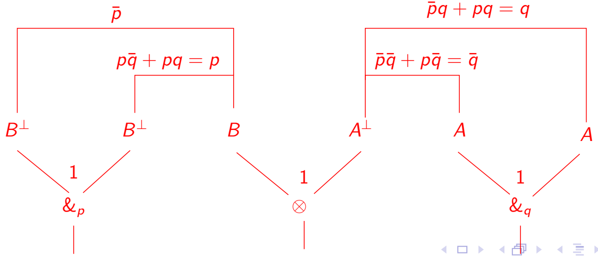
There is **a canonical surjection** from MALL SP to Polynomial PN

Example:

$$\dfrac{\dfrac{\dfrac{\bar{p}}{B^{\perp}, B} \qquad \dfrac{\dfrac{\bar{p}\bar{q}}{A^{\perp}, A} \quad \dfrac{\bar{p}q}{A^{\perp}, A}}{A^{\perp}, A \&_q A}\,\&}{B^{\perp}, B \otimes A^{\perp}, A \&_q A} \qquad \dfrac{\dfrac{\dfrac{p\bar{q}}{B^{\perp}, B} \quad \dfrac{p\bar{q}}{A^{\perp}, A}}{B^{\perp}, B \otimes A^{\perp}, A}\,\otimes \quad \dfrac{\dfrac{pq}{B^{\perp}, B} \quad \dfrac{pq}{A^{\perp}, A}}{B^{\perp}, B \otimes A^{\perp}, A}\,\otimes}{B^{\perp}, B \otimes A^{\perp}, A \&_q A}\,\&}{B^{\perp} \&_p B^{\perp}, B \otimes A^{\perp}, A \&_q A}\,\&_p$$

– a Polynomial PN is a sequent forest with weighted axioms

– replace parallel axioms $AX_1, AX_2, ..AX_n$ with weights $w_1, w_2, ..., w_n$, by a signle AX with weight $w = \sum_i^n w_i$.



It is **invariant** under the raising of the $⅋, \otimes, \oplus, \&$-rule over $\&$-rule:

Example:

$$
\dfrac{
\dfrac{
\dfrac{\dfrac{\bar{p}\bar{q}}{B^\perp, B} \quad \dfrac{\bar{p}\bar{q}}{A^\perp, A}}{B^\perp, B \otimes A^\perp, A}\otimes
\quad
\dfrac{\dfrac{\bar{p}q}{B^\perp, B} \quad \dfrac{\bar{p}q}{A^\perp, A}}{B^\perp, B \otimes A^\perp, A}\otimes
}{B^\perp, B \otimes A^\perp, A \&_q A}\&
\quad
\dfrac{
\dfrac{\dfrac{p\bar{q}}{B^\perp, B} \quad \dfrac{p\bar{q}}{A^\perp, A}}{B^\perp, B \otimes A^\perp, A}
\quad
\dfrac{\dfrac{pq}{B^\perp, B} \quad \dfrac{pq}{A^\perp, A}}{B^\perp, B \otimes A^\perp, A}
}{B^\perp, B \otimes A^\perp, A \&_q A}
}{B^\perp \&_p B^\perp, B \otimes A^\perp, A \&_q A}
$$

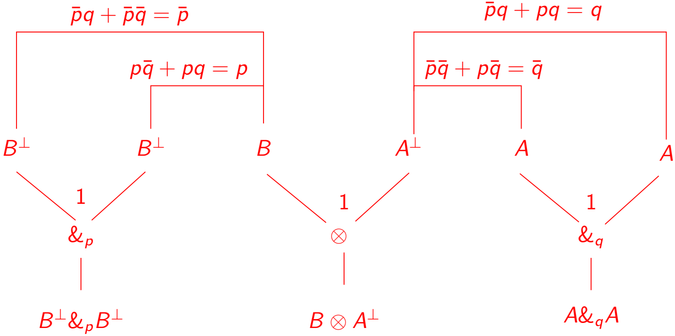
Interpretation: MALL SP $\mapsto$ Polynomial PN (8/8)

It is **invariant** under the raising of the $\parr, \otimes, \oplus, \&$-rule over $\&$-rule:

Example:

$$\dfrac{\dfrac{\dfrac{\dfrac{\bar p\bar q}{B^\perp, B} \quad \dfrac{\bar p\bar q}{A^\perp, A}}{B^\perp, B\otimes A^\perp, A}\otimes \quad \dfrac{\dfrac{\bar p q}{B^\perp, B} \quad \dfrac{\bar p q}{A^\perp, A}}{B^\perp, B\otimes A^\perp, A}\otimes}{B^\perp, B\otimes A^\perp, A\&_q A}\& \quad \dfrac{\dfrac{\dfrac{p\bar q}{B^\perp, B} \quad \dfrac{p\bar q}{A^\perp, A}}{B^\perp, B\otimes A^\perp, A} \quad \dfrac{\dfrac{pq}{B^\perp, B} \quad \dfrac{pq}{A^\perp, A}}{B^\perp, B\otimes A^\perp, A}}{B^\perp, B\otimes A^\perp, A\&_q A}}{B^\perp\&_p B^\perp, B\otimes A^\perp, A\&_q A}$$

maps to the same (previous) Polynomial PN:



Efficiency of the weight interpretations

- **monomial** and **polynomial** mapping are both efficient: P-time in the size of the SP.
- more efficient than **linkings mapping** (*HvG, 2003*): Exponential in the size of the SP.

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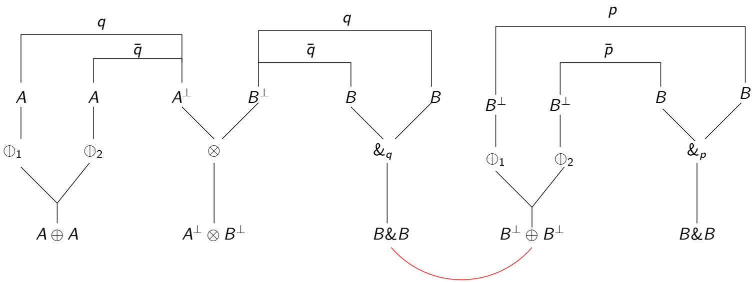
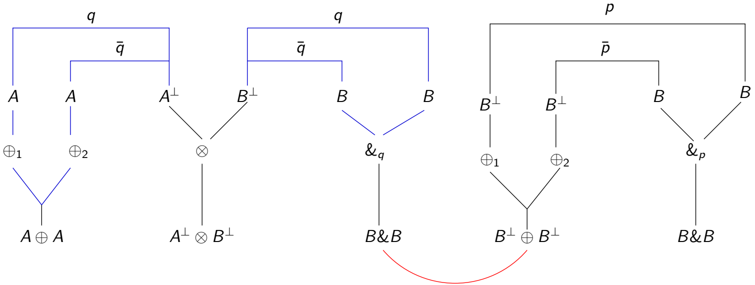
Global Cut-elimination with Monomial PS



reduces

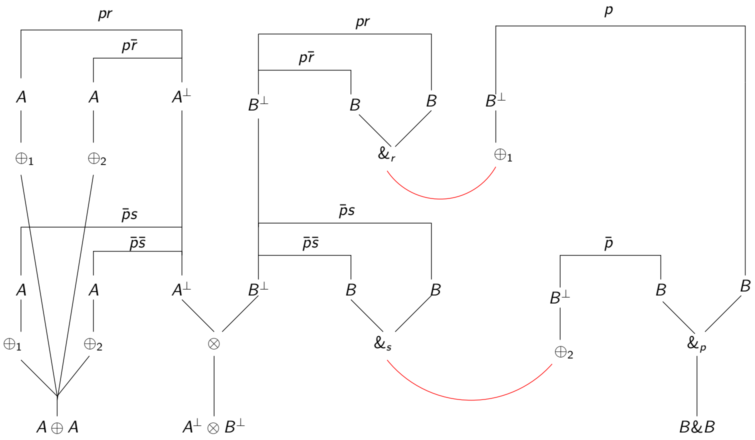
Global Cut-elimination with Monomial PS

via the **duplication** of the **dependency graph** of q (Maieli, 2007)



Global Cut-elimination with Monomial PS

to (we replace q by two news eigen-weights r and s):

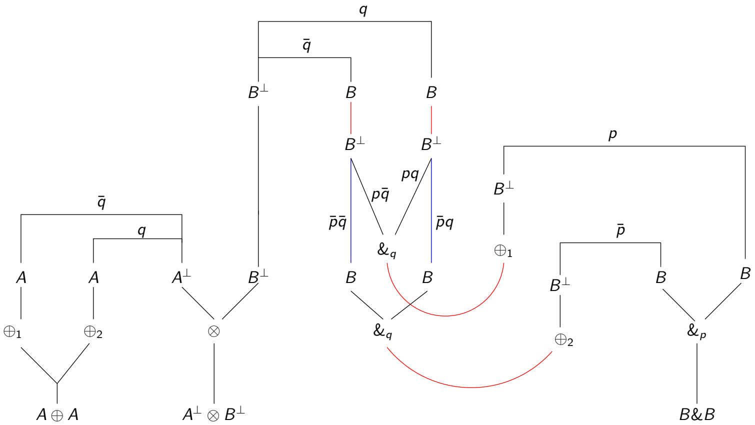


Local Cut-elimination with Monomial PS

or, via a **new dependence condition**

if L is a link depends on p then $w(L) \leq \sum_{i=1}^{n} w_i(\&_p)$

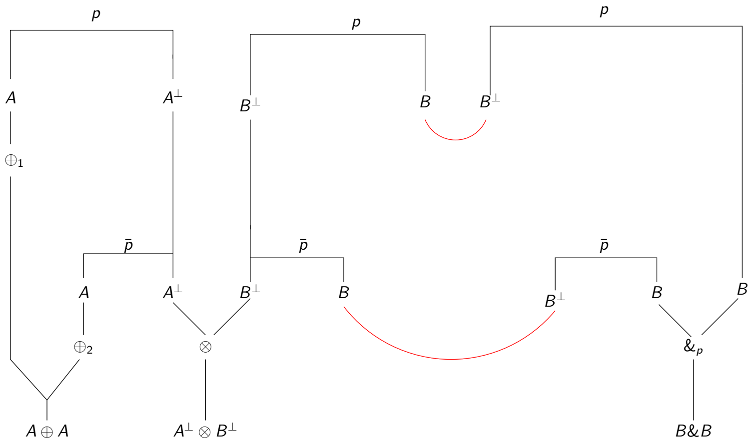
(Laurent-Maieli, 2008)



Cut-elimination with Monomial PS

finally, $\oplus_i/\&$ cuts reduce:

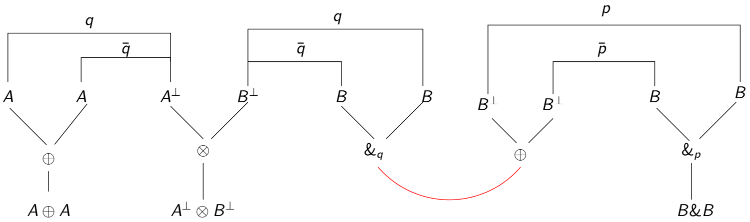
- globally, by erasing of slices $\bar{r}$ and s,
- locally, by erasing of slices $\bar{q}@p$ and $q@\bar{p}$)



Cut-elimination with Monomial PS

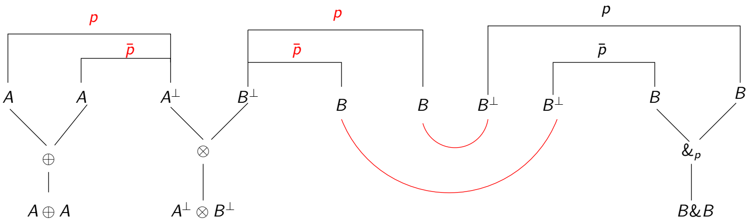
- both global and local cut elimination procedures are terminating and confluent;
- but with an unknown Complexity (P-time?).

Cut-elimination with Polynomial PS



reduces, to

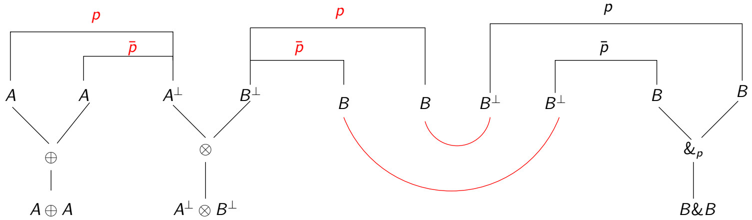
Cut-elimination with Polynomial PS



we replace $\begin{cases} \text{each occurrence of} & q & \text{by} & \sum_i^n w_i(B^\perp_{left}) = p \\ \text{each occurrence of} & \bar{q} & \text{by} & \sum_j^m w_j(B^\perp_{right}) = \bar{p} \end{cases}$

with w_i (resp., w_j) any weight belonging to an axiom with a literal conclusion occurring in the most left (resp., most right) $B^\perp$

Cut-elimination with Polynomial PS



we replace $\begin{cases} \text{each occurrence of } q \text{ by } \sum_i^n w_i(B^{\perp}_{left}) = p \\ \text{each occurrence of } \bar{q} \text{ by } \sum_j^m w_j(B^{\perp}_{right}) = \bar{p} \end{cases}$

with w_i (resp., w_j) any weight belonging to an axiom with a literal conclusion occurring in the most left (resp., most right) $B^{\perp}$

It is strong normalising (P-time) and confluent ($\sim$ Hughes, 2007)

Correctness Criterion for Monomial PN

(PS): the crucial point is the **dependence condition** ("&-boxing"):
if a link L depends on a variable p then $w(L) \leq w(\&_p)$.

(PN): every valuation induces a (unique) slice s.t. for every *switching* (obtained by mutilating one premise in each $\wp$ and by adding a jump from a $\&_p$-node to a node depending on p) is ACC.

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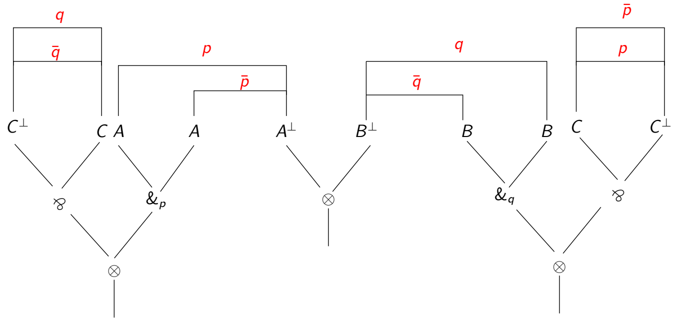
Example: a non correct PS



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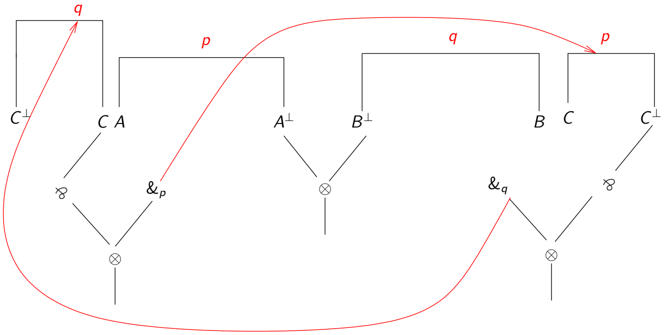
Example: there is a non-ACC switching with the *pq*-slice



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Checking Correctness and Sequentialization Complexity (P-time?)

Correctness Criterion for Polynomial PN (1/2)

(PS) – no dependence condition (weights are more liberal).
$+$ every valuation induces an unique (by $\oplus$-resolution) slice.

(PN) – Girard's criterion (by single switched slices) is not sufficient.

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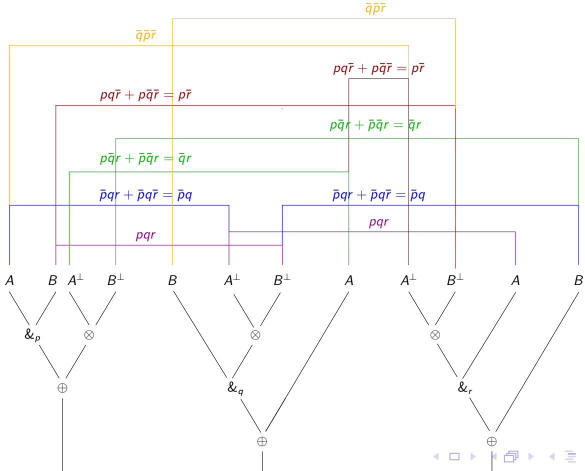
Gustave PS correct by single switched slices but non-sequentializable



Correctness Criterion for Polynomial PN (2/2)

Definition (HvG'03) : PN

(1) each slice is a MLL PN.

(2) every set of at least 2 slices *separates* (*toggles*) a & not belonging to any *switching cycle* [a cycle containing at most one switch edge (premise or jump edge) for each & and ⅋].

Checking Correctness and Sequentialization are P-time (Hughes,'07).

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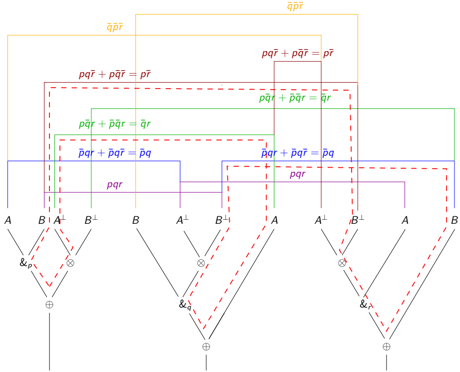
Gustave PS is, by (2), not correct



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conclusions

	Representation			*Cut-elimination*	
PN syntax	**P-time Correctness**	**P-time Translation**	**Abstraction**	**P-time**	**Confluence**
Monomial	?	linear	No	?	Yes
Polynomial	Yes	linear	Yes	Yes	Yes