Erratum to the paper “The relational model is injective for Multiplicative Exponential Linear Logic (without weakenings)”

Daniel de Carvalho and Lorenzo Tortora de Falco

In Definition 17, the set $b(v)$ has to contain only ports of the PS that are minimal in B_v w.r.t. the order relation $\leq_\Phi$, i.e. we have to add the following condition on the function b:

for any $v \in \mathcal{C}^!(\Phi)$, for any $p_1, p_2 \in b(v)$, we have $p_1 \leq_\Phi p_2 \Rightarrow p_1 = p_2$,

which yields the following definition:

Definition 17 *A* Proof-Structure (PS) *is a pair $R = (\Phi, b)$ where $\Phi \in$ **LPS** and b is a function $\mathcal{C}^!(\Phi) \to \mathfrak{P}(\mathsf{Auxdoors}(\Phi))$ such that for any $p \in \mathsf{Auxdoors}(\Phi)$, $\#_\Phi(p) = Card\{l \in \mathcal{C}^!(\Phi) \mid p \in b(l)\}$ and, for any $v \in \mathcal{C}^!(\Phi)$, for any $p_1, p_2 \in b(v)$, we have $p_1 \leq_\Phi p_2 \Rightarrow p_1 = p_2$. Proof-Structures are defined by induction on the number of* $!$-*cells: we ask that with every $v \in \mathcal{C}^!(\Phi)$ is associated a PS called* the box *of v (denoted by $\overline{\mathsf{B}}(R)(v)$) , and defined from the following subset B_v of $\mathcal{P}(\Phi)$:*

$$B_v = \{q \in \mathcal{P}(\Phi) \mid (\exists p \in \mathsf{P}^{\mathsf{aux}}_\Phi(v) \cup b(v))\ p \leq_\Phi q\}.$$

We ask that for $v, v' \in \mathcal{C}^!(\Phi)$ either $B_v \cap B_{v'} = \emptyset$ or $B_v \subseteq B_{v'}$ or $B_{v'} \subseteq B_v$.

*In order to define $\overline{\mathsf{B}}(R)(v)$ one first defines $\Psi \in$ **PLPS**, starting from two sets $\mathcal{L}_0$ and $\mathcal{P}_0$ and from two bijections $p_1 : \mathcal{L}_0 \to b(v)$ and $p_0 : \mathcal{L}_0 \to \mathcal{P}_0$, by setting:*

- $\mathcal{C}(\Psi) = \mathcal{L}_0 \uplus (\mathfrak{P}(\mathsf{C}_\Phi)(B_v) \setminus \mathfrak{P}(\mathsf{C}_\Phi)(b(v)))$;
  $\mathsf{t}_\Psi|_{\mathfrak{P}(\mathsf{C}_\Phi)(B_v)\setminus\mathfrak{P}(\mathsf{C}_\Phi)(b(v))} = \mathsf{t}_\Phi|_{\mathfrak{P}(\mathsf{C}_\Phi)(B_v)\setminus\mathfrak{P}(\mathsf{C}_\Phi)(b(v))}$ *and* $\mathsf{t}_\Psi(l) = ?$ *for every* $l \in \mathcal{L}_0$;
- $\mathcal{P}(\mathbb{C}(\Psi)) = (B_v \cup \{\mathsf{P}^{\mathsf{pri}}_\Phi(v)\}) \uplus \mathcal{P}_0$;
- $$\mathsf{C}_\Psi(p) = \begin{cases} \mathsf{C}_\Phi(p) & \textit{if } p \in B_v \setminus b(v); \\ l & \textit{if } p = p_1(l) \textit{ for } p \in b(v); \\ l & \textit{if } p = p_0(l) \textit{ for } p \in \mathcal{P}_0; \\ v & \textit{if } p = \mathsf{P}^{\mathsf{pri}}_\Phi(v); \end{cases}$$
- $$\mathsf{P}^{\mathsf{pri}}_\Psi(l) = \begin{cases} \mathsf{P}^{\mathsf{pri}}_\Phi(l) \textit{ if } l \notin \mathcal{L}_0; \\ p_0(l) \textit{ if } l \in \mathcal{L}_0; \end{cases}$$

- $P^{left}_{\Psi} = P^{left}_{\Phi}|_{\mathcal{C}^m(\Phi)\cap\mathfrak{P}(C_\Phi)(B_v)}$;
- $\#_\Psi(p) = Card\{w \in \mathcal{C}^!(\Phi) \cap \mathfrak{P}(C_\Phi)(B_v) \mid w \neq v$ *and* $p \in b(w)\}$;
- $\mathcal{I}(\Psi) = \emptyset$;
- $\mathcal{W}(\Psi) = \{\{p, q\} \in \mathcal{W}(\Phi) \mid p, q \in B_v\}$.

The box of v*, denoted by* $\overline{B}(R)(v)$*, is the pair* (Φ_v, b_v)*, where* Φ_v *is obtained from* Ψ *by eliminating the terminal link* v *(Definition 85) and* $b_v = b|^{\mathfrak{P}(Auxdoors(\Phi_v))}_{\mathcal{C}^!(\Phi_v)}$.

We set $LPS(R) = \Phi$, $b(R) = b$ *and we will write* the ports of R *(resp.* the cells of R*) meaning the ports of* Φ *(resp. the cells of* Φ*).*