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La logica dopo le macchine

Turing e gli altri

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outline

1. La logica dopo la macchina secondo Turing: la II macchina di Turing e gli studi sull'intelligenza meccanica
2. Come cambia la logica dopo le macchine: Gödel, Post, von Neumann e Wiener

Turing interprete di se stesso

1912-1954

Computabilità = calcolabilità effettiva

[…]"A function is effectively calculable if its values can be found by some purely mechanical process". We may take this statement literally, understanding by a **purely mechanical process one which could be carried out by a machine**. It is possible to give a mathematical description, in a certain normal form, of the structures of these machines. The development of these ideas leads to the author's definition of computable function, and to an **identification of computability with effective calculability.**

Turing 1939 in Copeland 2004: 150-151

La logica formale ha bisogno di intuizione

- In consequence of the impossibility of finding a **formal logic which wholly eliminates the necessity of using intuition**, we naturally turn to "non-constructive" systems of logic with which not all the steps in a proof are mechanical, some being intuitive […]
- What properties do we desire a non-constructive logic to have if we are to make use of it for the expression of mathematical proofs?
  - We want it to show quite clearly when a step makes use of intuition, and when it is purely formal.
  - The strain put on the intuition should be a minimum.
  - Most important of all, it must be beyond all reasonable doubt that the logic leads to correct results whenever the intuitive steps are correct.

Turing 1939 in Copeland 2004:193

Una visione hilbertiana 'meno radicale'

I think you take a much more radically Hilbertian attitude about mathematics than I do. You say 'if all this whole formal outfit is not about finding proofs which can be checked on a machine it's difficult to know what it is about'. When you say 'on a machine' do you have in mind that there is (or should be or could be, but has not been actually described anywhere) some fixed machine on which proofs are to be checked, and that the formal outfit is, as it were, about this machine. If you take this attitude (and it is this one that seems to me so extreme Hilbertian) there is little more to be said: we simply have to get used to the technique of this machine and resign ourselves to the fact that there are some problems to which we can never get the answer.

Turing's letter to M. H. A. Newman undated, but probably written in 1940 : KCCMA Turing's Papers: D2, Copeland 2004: 215

Varie macchine

If you think of various machines I don't see your difficulty. One imagines different machines allowing different sets of proofs, and by choosing a suitable machine one can approximate 'truth' by 'provability' better than with a less suitable machine, and can in a sense approximate it as well as you please. The choice of a proof checking involves intuition, […]

Turing's letter to Newman, *cit.*

La seconda Macchina di Turing

- Turing ha scritto la proposta per ACE (Automatic Computing Engine) nella seconda metà del 1945
- La macchina pratica era diversa da quella teorica a causa degli stringenti limiti fisici spazio-temporali
- ACE era simile alla Macchina Universale perché si basava su istruzioni ed era un dispositivo virtuale basato sulla simulazione
- Secondo Turing, tuttavia: "one will not be able to prove any result of the required kind [related to the equivalence of universal and practical machines], which gives any intellectual satisfaction"

(Turing 1948 in Copeland 2004: 416)

I limiti della macchina di Turing

Some years ago I was researching on what might now be described as **an investigation of the theoretical possibilities and limitations of digital computing machines**. I considered a type of machine which had a central mechanism, and an infinite memory which was contained on an infinite tape. This type of machine appeared to be sufficiently general. One of my conclusions was that the idea of a 'rule of thumb' process and a 'machine process' were synonymous. The expression 'machine process' of course means one which could be carried out by the type of machine I was considering.

Turing 1947 in Copeland 2004: 378-379

La II Macchina di Turing

Dispositivi non organizzati e automodificantesi

La nuova macchina e la logica

- The language in which one communicates with these machines, i.e. the language of instruction tables, forms a sort of symbolic logic. […] Actually one could communicate with these machines in any language provided it was an exact language […]
- Some attempts will probably be made to get the machine to do actual manipulations of mathematical formulae. To do so will require the development of a special logical system for the purpose. This system should resemble normal mathematical procedure closely, but at the same time should be as unambiguous as possible.

Turing 1947 in Copeland 2004:392

Turing scrive ad Ashby (1946?)

- *In working on the ACE I am more interested in the possibility of producing models of the action of the brain than in the practical applications to computing* […]
- *It would be quite possible for the machine to try out variations of behaviour and accept or reject them in the manner you describe and I have been hoping to make the machine to do this. This is possible because, without altering the design of the machine itself, it can, in theory at any rate, be used as a model of any other machine, by making it remember a suitable set of instructions.*

Turing's letter to Ashby, undated but probably written in 1946

Il cervello come un 'continuous controlling'

We shall mainly be concerned with discrete controlling machinery. As we have mentioned, brains very nearly fall into this class, and there seems every reason to believe that they could have been made to fall genuinely into it without any change in their essential properties. However, the property of being 'discrete' is only an advantage for the theoretical investigator, and serves no evolutionary purpose, so we could not expect Nature to assist us by producing truly 'discrete' brains.

Turing 1948 in Copeland 2004: 412-413

Macchine non organizzate

- We might instead consider what happens when we make up a machine in a comparatively unsystematic way from some kind of standard components. We could consider some particular machine of this nature and find out what sort of things it is likely to do. Machine which are largely random in their construction in this way will be called 'unorganised machine' (Turing 1948: 416)
- I would like to investigate other types of unorganised machine, and also to try out organising methods that would be more nearly analogous to our 'methods of education' (Turing 1948: 428)

La corteccia come una macchina non organizzata

All of this suggests that the cortex of the infant is an unorganized machine, which can be organised by suitable interfering training. The organising might result in the modification of the machine into a universal machine or something like it. This would mean that the adult will obey orders given in appropriate language, even if they were very complicated; he would have no common sense, and would obey the most ridiculous orders […]. Creatures not unlike this can really be found, but most people behave quite differently under many circumstances. However the resemblance to a universal machine is still very great, and suggest to us that the step from the unorganised infant to a universal machine is one which should be understood. When this has been mastered we shall be in a far better position to consider how the organising process might have been modified to produce a more normal type of mind

Turing 1948 in Copeland 2004 : 424

Il ruolo del caso e dell'organizzazione

The configurations of the machine are described by two expressions, which we may call the character-expression and the situation-expression. […] The character may be subject to some random variation. Pleasure interference has a tendency to fix the character i.e. towards preventing it changing, whereas pain stimuli tend to disrupt the character, causing features which had become fixed to change, or to become again subject to random variation.

Turing 1948 in Copeland 2004: 425

Caratteristiche della macchina non organizzata e automodificantesi

- Può commettere errori: non è completamente affidabile e non può essere controllata solo dalla logica
- Può offrire diversi output rispetto agli stessi problemi perché apprende dall'esperienza passata
- Il mondo esterno esercita un ruolo rilevante nel processo di training
- L'acquisizione di conoscenza è causata dall'effetto congiunto di inferenza esterna (che include anche l'interazione sociale) automodificazione, casualità e altre strategie di apprendimento, e non è un'attività solitaria come accade nella logica

Disciplina e iniziativa

So far we have been considering only discipline. To convert a brain or machine into a universal machine is the extremest form of discipline. Without something of this kind one cannot set up proper communication. But discipline is certainly not enough in itself to produce intelligence. That which is required in addition we call initiative. […] Our task is to discover the nature of this residue as it occurs in man, and to try and copy it in machines.

Turing 1948 in Copeland 2004:429

Ricerca genetica per l'intelligenza

There is a genetical or evolutionary search by which a combination of genes is looked for, the criterion being survival value. The remarkable success of this search confirms to some extent the idea that intellectual activity consists mainly of various kinds of search.

Turing 1948 in Copeland 2004: 431

Inadeguatezza della 'ragione' senza senso comune

The results which have been described in this article are mainly of a negative character, setting certain bounds to what we can hope to achieve purely by reasoning. These, and some other results of mathematical logic may be regarded as going some way towards a demonstration, within mathematics itself, of the inadequacy of 'reason' unsupported by common sense

Turing 1954 in Copeland 2004:595

L'embriologia e il cervello

- *I am afraid I am very far from the stage where I feel inclined to start asking any anatomical questions [about brain] […] At present I am not working on the problem at all but on my mathematical theory of embryology, […]. I am really doing this now because it is yielding more easily to treatment. I think it is not altogether unconnected with the other problem [the brain storage]. The brain structure has to be one which can be achieved by genetical embryology mechanism, and I hope that this theory that I am now working on, may make clearer what restrictions this really implies. What you tell me about growth of neurons under stimulations, is very interesting […]. It suggests means by which the neurons may be made to grow so as to form a particular circuit rather than to reach a particular place.* *Turing's letter to Young 8/2/1951 – K78 Tuiring's Papers KCC –Modern archives*

Macchine non organizzate, logica e intelligenza

- La posizione di Turing rispetto alla visione hilbertiana comincia a cambiare almeno a partire dal 1939
- Fin dal principio della costruzione di ACE era interessato alla costruzione di un dispositivo intelligente
- L'obiettivo dell'intelligenza meccanica lo convince della centralità di elementi come l'organizzazione, la randomizzazione, la flessibilità agli errori, le strategie per l'apprendimento
- È possibile che i suoi interessi per l'embriologia siano emersi dal progetto stesso dell'intelligenza meccanica: dal desiderio di comprendere il funzionamento dello sviluppo della vita e dell'organizzazione del cervello, con l'obiettivo successivo di simularli attraverso una nuova generazione di macchine

Gödel, Post, Von Neumann e Wiener sulla logica dopo la macchina

Anche nel dibattito con Turing

Kurt Gödel (1906-1978)

Gödel discute il risultato di Turing/1

- This importance [of Turing's computability] is largely due to the fact that with this concept one has for the first time succeeded in giving an absolute definition of an interesting epistemological notion, i.e. one not depending on the formalism chosen.
- By a kind of miracle it is not necessary to distinguish orders, and the diagonal procedure does not lead outside the defined notion.

Gödel 1946/1990: 150

Gödel discute il risultato di Turing/2

- Turing's work gives an analysis of the concept of '**mechanical procedure**' (alias 'algorithm' or 'computation procedure' or 'finite combinatorial procedure'). **This concept is shown to be equivalent with that of a 'Turing machine**'. A formal system can simply be defined to be any mechanical procedure for producing formulas, called provable formulas. […]

Gödel 1964: 72

Gödel ritiene che i sistemi formali siano procedure meccaniche…ma

- This meaning [the fact that a finite procedure is identified with a mechanical procedure], however is required by the **concept of formal system**, whose essence it is that **the reasoning is completely replaced by mechanical operations on formulas**.
- (Note that the question of whether there exist finite non-mechanical procedures not equivalent with any algorithm, has nothing whatsoever to do with the adequacy of the definition of 'formal system' and of mechanical procedure).

Gödel *Ibidem*

Gödel contro Turing: procedure mentali vs meccaniche/1

- Turing gives an argument which is supposed to show that mental procedures cannot go beyond mechanical procedures.
- What Turing disregards completely is the fact that the ***mind in its use is not static, but constantly developing***, i.e. that we understand abstract terms more and more precisely as we go on using them, and that more and more abstract terms enter the sphere of our understanding.

Gödel 1972a/1990: 306

Gödel contro Turing: procedure mentali vs meccaniche/2

- Therefore, although at each stage of the number and precision of the abstract terms at our disposal may be finite, both (and, **therefore also Turing's number of *distinguishable states of mind*) may *converge toward infinity*** in the course of the application of the procedure.

Gödel *ibidem*

Cosa pensava Gödel?

- Era convinto che la definizione di computabilità effettiva fosse equivalente a quella di Turing calcolabilità
- Non era convinto che tutte le procedure finite potessero essere esaurite dalle procedure meccaniche
- Cercava metodi per identificare procedure costruttive sebbene non meccaniche: ad es. quelle sviluppate nella mente umana
- Si tratta di capacità di astrazione, di progressiva precisione e di successione dinamica,(di mente infinita?): non è facile riprodurre gli stati mentali nella simbologia della MT
- Al cuore di questa visione c'è la concezione della complessità, centrale anche per la posizione di von Neumann sugli automi

Una piccola curiosità

- Una lettera di Gödel a von Neumann del 20 marzo 1956 nella quale si anticipa la questione P=?PN, con Gödel che sostiene ingenuamente la sua fiducia nell'uguaglianza dei diversi tipi di soluzione di problemi
- Questo è il sito dove si trovava originariamente il testo inglese e l'originale tedesco.
- Attualmente la versione inglese si trova qui
- Per rivedere la pagina originale nella quale era riprodotta la lettera si può ricorrere al sito archive.org a questo indirizzo

Emil Post (1897-1954)

L'ipotesi di Church come una legge naturale

- We offer this conclusion at the present moment as a ***working hypothesis***. And to our mind such is Church's identification of effective calculability with recursiveness. […]
- The success of the above program would, for us, change this hypothesis not so much to a definition or to an axiom but to a ***natural law***.

Post 1936/1965: 291

La creatività del logico

- A **complete symbolic logic is impossible**
- This is an iconoclastic result from the formal logician's point of view since it means that logic must not only in some parts of its description […], but in its very operation be **informal**
- The **Logical Process is Essentially Creative**
- We see that a machine would never give a complete logic; for once the machine is made we could prove a theorem it does not prove

Post 1941/1965: 416-417

John Von Neumann (1903-1957)

Le domande di von Neumann

- Com'è possibile costruire sistemi affidabili che usino componenti inaffidabili?
- Che tipo di organizzazione logica sarebbe sufficiente per un automa capace di riprodurre se stesso?

La logica in termini di automi

- I risultati di Gödel e Turing permettono di guardare alla logica in termini di automi
- Le proposizioni logiche possono essere rappresentate come reti elettriche o sistemi nervosi idealizzati
- Ma c'è una grande differenza: il ruolo del tempo che è centrale negli automi mentre è escluso dalla logica
- Il tempo implica la gestione delle risorse, la loro disponibilità e l'analisi sulla dimensione dei calcoli

La teoria degli automi (TA)

- Si pone problemi legati a:
  - Organizzazione
  - Strutture
  - Linguaggio
  - Informazione
  - Controllo
- La TA ha delle relazioni con teoria della comunicazione e ingegneria del controllo da un lato e con la biologia dall'altro

I problemi aperti dalla logica degli automi

- La distinzione tra digitale e analogico
- L'affidabilità di dispositivi costituiti da componenti instabili e la gestione degli errori
- Il ruolo della complessità
- La differenza tra automi naturali che utilizzano maggiormente le procedure parallele e automi artificiali che tendono ad essere più seriali nelle operazioni

Il funzionamento del sistema nervoso

- Yet the nervous system seems to be somewhat more flexibly designed [than the multiplexed automaton]. Also, its "digital" (neural) operations **are rather freely alternating with "analog"** (hormonal) processes in their complete chain of causation. Finally, the whole **logical pattern** of the nervous system **seems to deviate** in certain important traits qualitatively and significantly **from our ordinary mathematics and mathematical-logical** mode of operations

Von Neumann 1952: 87-88

Le inadeguatezze della logica

- [Logic] deals with rigid, all-or-none concepts, and has very little contact with the continuous concept of the real or of the complex numbers, that is, with mathematical analysis. […] Thus **formal logic is, by the nature of its approach, cut off from the best cultivated portions of mathematics**, and forced onto the most difficult part of the mathematical terrain, into combinatorics […]
- In studying the functioning of automata, it is clearly **necessary to pay attention to a circumstance which has never before made its appearance in formal logic**

Von Neumann 1948/1961:303

Gestire gli errori

Natural organisms are constructed to make errors as inconspicuous, as harmless, as possible. Artificial automata are designed to make errors as conspicuous, as disastrous as possible. […] Natural organisms are sufficiently well conceived to be able to operate even when malfunctions have set in. […]

Von Neumann 1948/1961: 306

La logica degli automi

The logic of automata will differ from the present system of formal logic in two relevant respects:

1. The **actual length of "chains of reasoning", that is, of the chains of operations, will have to be considered**.
2. The operations of **logic** […] will have to be treated **by procedures which allow exceptions (malfunctions) with low but non-zero probabilities**. All of this will lead to theories which are much less rigidly of an all-or-none nature than past and present formal logic. They will be of a much less combinatorial, and much more analytical, character.

Von Neumann 1948/1961: 304

La logica dell'analogia

- Nobody would attempt to describe and define within a practical amount of space the general concept of analogy which dominates our interpretation of vision […]
- We are dealing here with parts of logic with which we have practically no past experience. **The order of complexity is out of all proportion to anything we have ever known**
- We have no right to assume that the logical notations and procedures used in the past are suited to this part of the subject […]
- It is therefore, not at all unlikely that it is futile to look for a precise logical concept, that is, for a precise verbal description of "visual analogy"

Von Neumann 1948/1961: 311

Rigore assoluto e precisione

- It is perfectly true that in all mathematical problems the answer is required with absolute rigor, with absolute reliability. This may, but need not, mean that it is also required with absolute precision.
- In most problems for the sake of which computing machines are being built […] **the precision that is wanted is quite limited**. That is the data of the problem are only given to a limited precision, and the result is only wanted to a limited precision. This is quite compatible with absolute mathematical rigor […]

Von Neumann 1948/1961: 324-325

La teoria della complicazione: ancora organizzazione

- […] "complication" on its lower lever is probably degenerative, that is, that every automaton that can produce other automata will only be able to produce less complicated ones.
- There is, however, a certain minimum level where this degenerative characteristic ceases to be universal. At this point automata which can reproduce themselves, or even construct higher entities, become possible.
- This fact, that **complication**, as well as **organization**, **below a certain minimum level is degenerative, and beyond that level can become self-supporting and even increasing**, will clearly play an important role in any future theory of the subject

Von Neumann 1948/1961:318

Il progetto finale di Von Neumann

- I suspect that a deeper mathematical study of the nervous system […] **will affect our understanding of the aspects of mathematics itself that are involved**. In fact, it may alter the way in which we look on mathematics and logics proper.

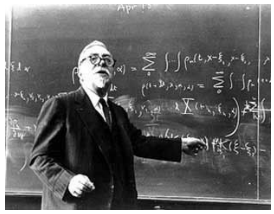
Von Neumann 1958:2

Logica,matematica e storia

- Just as languages like Greek or Sanskrit are historical fact and not absolute logical necessities, it is only reasonable to assume that **logics and mathematics are similarly historical, accidental forms of expression**. They may have essential variants, i.e. they may exist in other forms than the ones to which we are accustomed. […]
- We have now accumulated sufficient evidence to see that **whatever language the central nervous system is using**, it is characterized **by less logical and arithmetical depth** than what we are normally used to.

Von Neumann 1958:81

La Cibernetica e l'approccio di Wiener alle macchine



La macchina calcolatrice ideale

- Thus the computing machine must be a logical machine as well as an arithmetic machine and must combine contingencies in accordance with a systematic algorithm

Wiener 1948: 118

- It is by no means trivial to consider the light cast on logic by such machines, both natural and artificial. […] the study of logic must reduce to the study of the logical machine, whether nervous or mechanical, with **all its non-removable limitations and imperfections.**

Wiener 1948: 124-125

Logica e psicologia

- It may be said […] that this reduces logic to psychology[…] Psycology contains much that is foreign to logic, but […] any logic which means anything to us can contain nothing which the human mind – and hence the human nervous system – is unable to encompass. ***All logic is limited by the limitation of the human mind when it is engaged in that activity known as logical thinking.***

Wiener 1948: 125

La macchina e la logica umana

- We thus see that the **logic of the machine resembles human logic**, and, following Turing, we may employ it to **throw light on human logic**. Has the machine a more eminently human characteristic as well – **the ability to learn**? To see that it may well have even this property, let us consider two closely related notions: that of the **association of ideas** and that of the **conditioned reflex**.

Wiener 1948: 126

Riflesso condizionato e comunicazione

- Conditioned reflex is a learning mechanism […] all that is needed is that the inducements or punishments used have, respectively a positive and a negative affective tone […]
- Another point of considerable interest is that such a mechanism involves a certain set of messages which go out generally into the nervous system, to all elements which are in a state to receive them

Wiener 1948: 129

La battaglia per la conoscenza

- […] I could not bring myself to believe in the existence of a closed set of postulates for all logic, leaving no room for any arbitrariness in the system defined by them […].
- To me, logic and learning and all mental activity have always been incomprehensible as a complete and closed picture and have been understandable only as a process by which man puts himself *en rapport* with his environment. It is the battle for learning which is significant, and not the victory.

Wiener 1956: 324

Logica e ingegneria

- The *machina ratiocinatrix* is nothing but the *calculus ratiocinator* of Leibniz with an engine in it; and just as modern mathematical logic begins with this calculus, so **it is inevitable that its present engineering development should cast a new light on logic**.
- The science of today is operational; it considers every statement as essentially concerned with possible experiments or observable processes.

Wiener 1948: 125

Osservazioni finali…

- Turing, von Neumann e Wiener condividono l'ipotesi che un rinnovo della logica fosse necessario per costruire macchine intelligenti o che si autoriproducano
- Se le macchine devono emulare l'attività del cervello, devono usare una nuova logica, a causa della parziale irriducibilità dei fenomeni continui che sono coinvolte dal funzionamento del sistema nervoso
- Turing e von Neumann sembrano essere compagni nell'attribuire grande importanza alle procedure casuali e alle strategie di organizzazione che possono essere realizzate con vari metodi

… osservazioni finali

- Von Neumann era affascinato dal concetto di complicazione che sopra una certa soglia diventava virtuoso invece di essere degenerativo
- Wiener progettava a partire dalle idee di meccanismo di feedback e capacità di comunicazione
- Sorprendentemente tutti questi elementi sono al centro della ricerca attuale di punta su teoria delle reti, A-life, quantum computing, bioinformatica, trattabilità etc.

…ultime osservazioni

- La logica subisce un grande cambiamento nell'incontro con le macchine,
- Le prospettive di rinnovamento vanno nella direzione di accogliere il tempo tra le variabili da modulare nel processo
- L'organizzazione dovrebbe svolgere un ruolo significativo insieme con la complicazione (o forse si dovrebbe dire gestione della complessità)
- Il progetto di Turing,von Neumann e Wiener è senz'altro simile
- Ma anche studiosi di logica più tradizionalisti sembrano andare nella stessa direzione epistemologica, sebbene non attribuiscano alle macchine la stessa importanza degli altri

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