Erratum to the paper “The relational model is injective for Multiplicative Exponential Linear Logic (without weakenings)”

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In Definition 17, the set \( b(v) \) has to contain only ports of the PS that are minimal in \( B_v \) w.r.t. the order relation \( \leq \), i.e. we have to add the following condition on the function \( b \):

for any \( v \in C'(\Phi) \), for any \( p_1, p_2 \in b(v) \), we have \( p_1 \leq \Phi p_2 \Rightarrow p_1 = p_2 \),

which yields the following definition:

**Definition 17** A Proof-Structure (PS) is a pair \( R = (\Phi, b) \) where \( \Phi \in LPS \) and \( b \) is a function \( C'(\Phi) \rightarrow \mathcal{P}(\text{Auxdoors}(\Phi)) \) such that for any \( p \in \text{Auxdoors}(\Phi) \), \( \#_\Phi(p) = \text{Card}\{l \in C'(\Phi) | p \in b(l)\} \) and, for any \( v \in C'(\Phi) \), for any \( p_1, p_2 \in b(v) \), we have \( p_1 \leq \Phi p_2 \Rightarrow p_1 = p_2 \). Proof-Structures are defined by induction on the number of \( ! \)-cells: we ask that with every \( v \in C'(\Phi) \) is associated a PS called the box of \( v \) (denoted by \( B(R)(v) \)), and defined from the following subset \( B_v \) of \( \mathcal{P}(\Phi) \):

\[
B_v = \{ q \in \mathcal{P}(\Phi) | (\exists p \in P^{aux}_\Phi(v) \cup b(v)) \ p \leq \Phi q \}.
\]

We ask that for \( v, v' \in C'(\Phi) \) either \( B_v \cap B_{v'} = \emptyset \) or \( B_v \subseteq B_{v'} \) or \( B_{v'} \subseteq B_v \).

In order to define \( B(R)(v) \) one first defines \( \Psi \in \text{PLPS} \), starting from two sets \( L_0 \) and \( P_0 \) and from two bijections \( p_1 : L_0 \rightarrow b(v) \) and \( p_0 : L_0 \rightarrow P_0 \), by setting:

- \( C(\Psi) = L_0 \sqcup (\mathcal{P}(C_\Phi)(B_v) \setminus \mathcal{P}(C_\Phi)(b(v))) \)
- \( \mathcal{P}(\mathcal{C}(\Psi)) = (B_v \cup \{ P^{\psi!}_\Phi(v) \}) \sqcup P_0 \)
- \( \mathcal{C}_\Psi(p) = \begin{cases} C_\Phi(p) & \text{if } p \in B_v \setminus b(v); \\ l & \text{if } p = p_1(l) \text{ for } p \in b(v); \\ l & \text{if } p = p_0(l) \text{ for } p \in P_0; \\ v & \text{if } p = P^{\psi!}_\Phi(v); \end{cases} \)
- \( P^{\psi!}_\Psi(l) = \begin{cases} P^{\psi!}_\Phi(l) & \text{if } l \notin L_0; \\ p_0(l) & \text{if } l \in L_0; \end{cases} \)
\( P_{\Psi}^\text{left} = P_{\Psi}^\text{left} \mid C_{m(\Phi)} \cap C_{\Phi}(B_v) \):

- \#_{\Psi}(p) = \text{Card}\{w \in C^1(\Phi) \cap P_{\Phi}(B_v) \mid w \neq v \text{ and } p \in b(w)\}:
- \mathcal{I}(\Psi) = \emptyset:
- \mathcal{W}(\Psi) = \{\{p, q\} \in \mathcal{W}(\Phi) \mid p, q \in B_v\}.

The box of \( v \), denoted by \( B(R)(v) \), is the pair \((\Phi_v, b_v)\), where \( \Phi_v \) is obtained from \( \Psi \) by eliminating the terminal link \( v \) (Definition 85) and \( b_v = b|_{C(\Phi_v)} \).

We set \( LPS(R) = \Phi, b(R) = b \) and we will write the ports of \( R \) (resp. the cells of \( R \)) meaning the ports of \( \Phi \) (resp. the cells of \( \Phi \)).